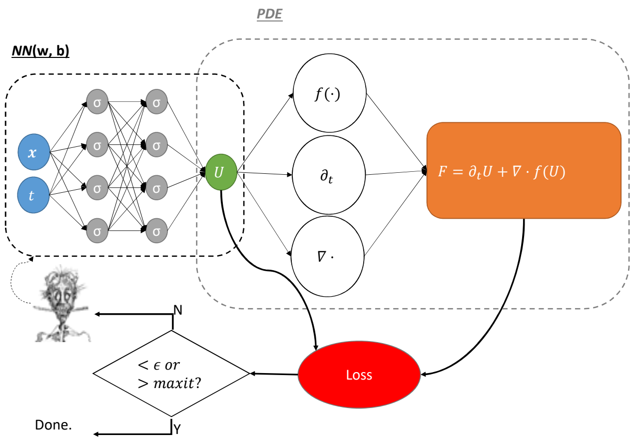
Time-parallel/fractional/adaptive activation function based physics-informed neural networks for solving transient PDEs

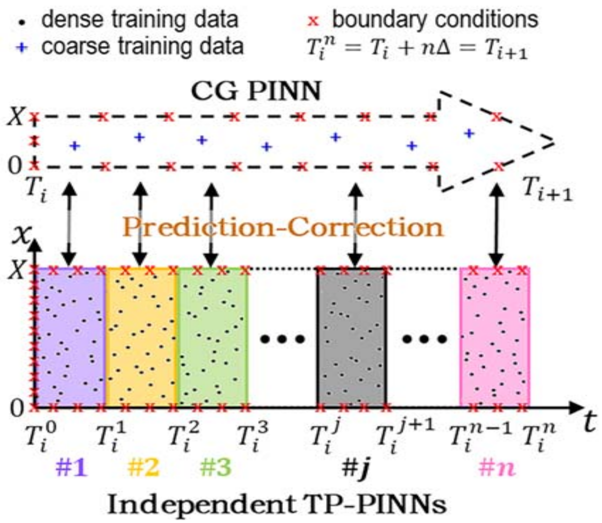
Guo fei Pang, Xuhui Meng, Ameya D. Jagtap, Zhen Li, Dongkun Zhang, and George Em Karniadakis
Division of Applied mathematics, Brown University *E-mail: {guofei_pang, xuhui_meng, ameya_jagtap}@brown.edu*

Physics-informed neural networks (PINNs)

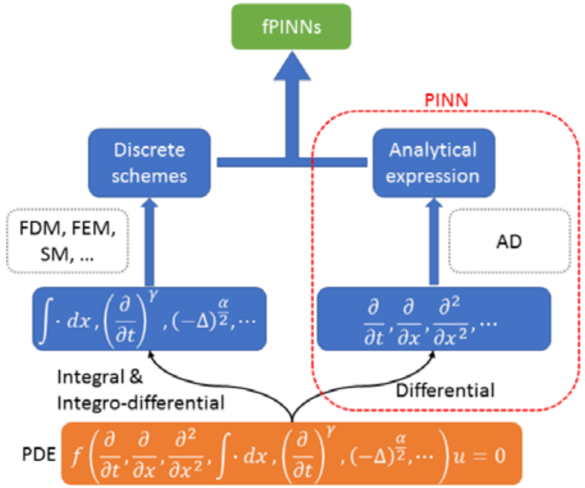
forward and inverse PDEs



Time-parallel PINNs



fPINNs – Fractional PINNs

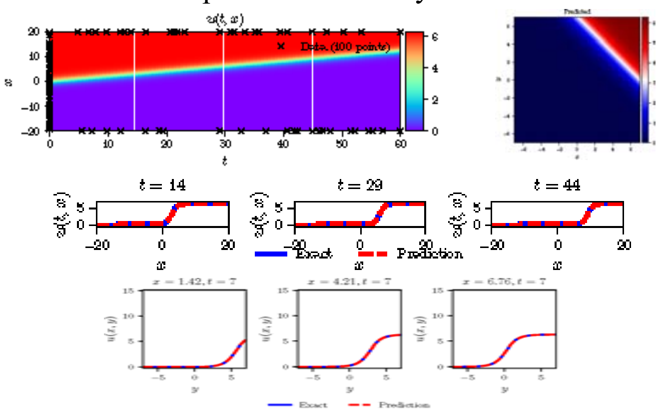


Sine-Gordon equation

$$\partial_t^2 \boldsymbol{u} - \boldsymbol{\Delta u} - \sin(\boldsymbol{u}) = 0$$

$$\boldsymbol{u}(\boldsymbol{x}, 0) = f(\boldsymbol{x}), \partial_t \boldsymbol{u}(\boldsymbol{x}, 0) = g(\boldsymbol{x})$$

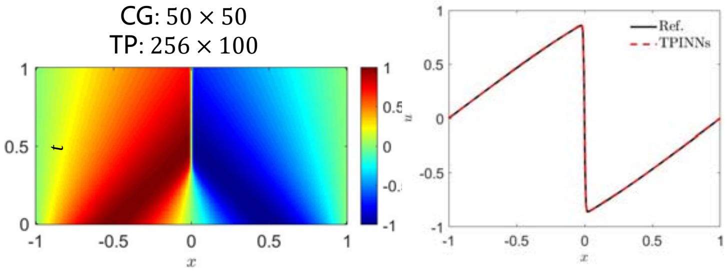
With extrapolation boundary conditions



Burgers equation

$$\partial_t u + u\partial_x u - \nu\partial_x^2 u = 0$$
$$u(x, 0) = -\sin(-\pi x)$$
$$u(-1, t) = u(1, t) = 0$$

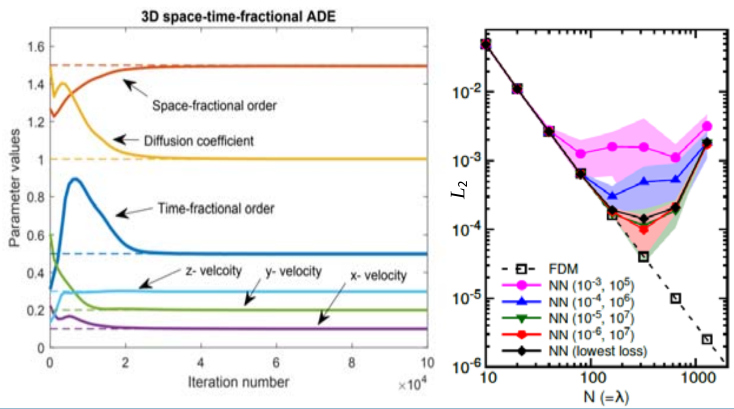
$$CG\!: \ \partial_t u + u\partial_x u - \nu_c \partial_x^2 u = 0$$
$$TP\!: \ \partial_t u + u\partial_x u - \nu\partial_x^2 u = 0$$
$$\nu = 0.01/\pi$$



	$\nu_c = 3\nu$	$\nu_c = 5\nu$	$\nu_c = 10\nu$
Iterations #	2	3	6

Parameter identification for 3D ADE

$$\frac{\partial^\gamma u(\boldsymbol{x},t)}{\partial t^\gamma} = -c(-\Delta)^{\alpha/2}u(\boldsymbol{x},t) - \boldsymbol{v}\cdot\nabla u(\boldsymbol{x},t) + f_{BB}(\boldsymbol{x},t), \quad \boldsymbol{x}\in\Omega\subset\mathbb{R}^D, t\in(0,T],$$
$$u(\boldsymbol{x},t) = 0, \quad \boldsymbol{x}\in\partial\Omega,$$
$$u(\boldsymbol{x},0) = g(\boldsymbol{x}), \quad \boldsymbol{x}\in\Omega.$$



References: 1. Z. Li, X. Meng, et al, Time-parallel PINNs, In preparation. 2. G. Pang, L, Lu, et al, fPINNs, SISC, 2019. 3. A. D. Jagtap, G. E. Karniadakis, Adaptive activation functions accelerate convergence in deep and physics-informed neural networks